

CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Level

FURTHER MATHEMATICS

9231/02

Paper 2

May/June 2003

3 hours

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Where a numerical value is necessary, take the acceleration due to gravity to be 10 m s^{-2} .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The use of a calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

You are reminded of the need for clear presentation in your answers.

This document consists of **5** printed pages and **3** blank pages.



- 1 A point P moves on the x -axis so that its x -coordinate, in metres, at time t seconds is given by

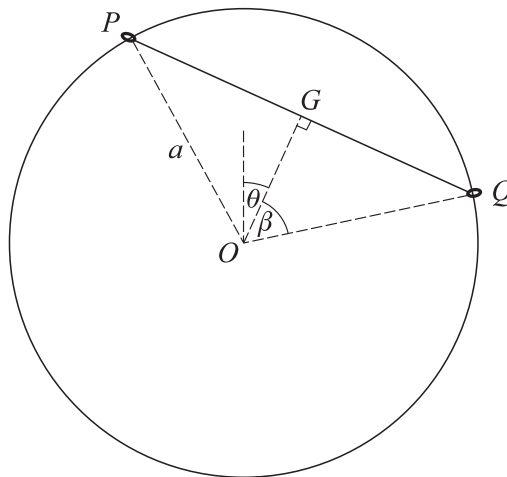
$$x = 12(1 - \sin 3t) + 5 \cos 3t.$$

(i) Show that $\frac{d^2x}{dt^2} = -9(x - 12)$. [1]

- (ii) Define *simple harmonic motion*, and show that P is describing this motion about a fixed point A . Give the x -coordinate of A , and find the period and amplitude of the motion of P . [4]

- 2 Two smooth uniform spheres, A and B , are of the same size, but A has twice the mass of B . They are placed at rest on a smooth horizontal table and A is projected directly towards B with speed U . The spheres collide, and then B strikes a barrier perpendicular to its path, rebounds, and hits A again. Find the velocities of A and B immediately after this second collision between A and B , given that the coefficient of restitution is $\frac{1}{2}$ at all three impacts, and neglecting air resistance. [7]

3



Two small rings, P and Q , each of mass m , are threaded on a smooth circular hoop of radius a and centre O , fixed in a vertical plane. P and Q are joined by a light inextensible string, and, when it is taut, the angle $POQ = 2\beta$. The mid-point of the string is G , and OG makes an angle θ with the upward vertical at time t (see diagram). The system is slightly displaced from its position of equilibrium in which $\theta = 0$. By considering the total energy of the system, show that, in the subsequent motion while the string is taut, the angular speed $\dot{\theta}$ of the system is given by

$$a\dot{\theta}^2 = 2g \cos \beta (1 - \cos \theta). \quad [3]$$

Consideration of the transverse component of the acceleration of P gives the equation

$$T \cos \beta - mg \sin(\beta - \theta) = ma\ddot{\theta},$$

where T is the tension in the string. State briefly what is represented by each term on the left hand side of this equation. [2]

Write down a similar equation for the transverse motion of Q , and hence, or otherwise, show that

$$T = mg \tan \beta \cos \theta. \quad [4]$$

3

4

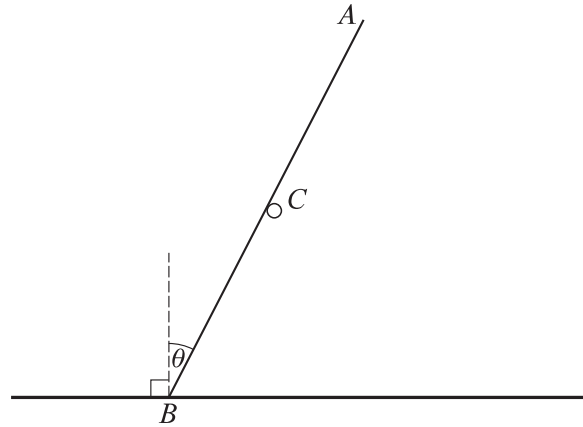


Fig. 1

A uniform rod AB , of weight W and length $2l$, rests against a smooth peg C with its end B on a rough horizontal floor. The rod rests in equilibrium at an angle θ with the vertical, and with C at the mid-point of AB (see Fig. 1). The coefficient of friction at B is μ . Show that $\mu \geq \tan \theta$. [5]

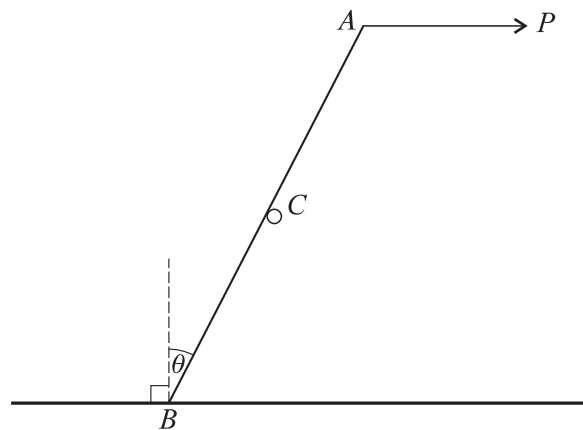


Fig. 2

A horizontal force is now applied at A , in the vertical plane through AB , and to the right (see Fig. 2). The magnitude P of this force is increased until the end B slides to the left. In the case when $\tan \theta = \frac{3}{4}$ and $\mu = \frac{5}{6}$, find the value of P when sliding begins. [6]

5 A rigid body of mass M consists of four equal uniform rods joined at their ends to form a square $ABCD$ of side $2a$. Show that the moment of inertia of this body about an axis

- (i) through its centre perpendicular to its plane is $\frac{4}{3}Ma^2$, [2]
- (ii) through A perpendicular to its plane is $\frac{10}{3}Ma^2$, [2]
- (iii) through A parallel to BD is $\frac{8}{3}Ma^2$. [4]

The body can rotate freely about a horizontal axis through A parallel to BD . The body is held with C vertically above A , and is slightly displaced. Neglecting air resistance, find the angular speed of the body when AC is horizontal. [3]

- 6 The approximate land areas (in suitable units) and populations (in millions) of six randomly selected Asian countries are given in the table.

| | | | | | | |
|--------------------|-----|------|------|-----|------|-------|
| Area (x) | 6.0 | 10.5 | 1.7 | 0.6 | 2.5 | 3.0 |
| Population (y) | 2.3 | 16.7 | 23.2 | 5.8 | 18.0 | 130.0 |

$$[\Sigma x = 24.3, \Sigma x^2 = 164.75, \Sigma y = 196.0, \Sigma y^2 = 18\,080.06, \Sigma xy = 667.07.]$$

Find the product moment correlation coefficient between x and y . [3]

Comment on the advisability of using the regression line of y on x to estimate the population of a seventh Asian country with a known land area. [2]

- 7 An official in a cycle race notes the number of cyclists passing her in each of 60 random one-minute intervals. Her results are given in the table.

| | | | | | | |
|--------------------|----|----|----|----|---|-----------|
| Number of cyclists | 0 | 1 | 2 | 3 | 4 | 5 or more |
| Frequency | 10 | 12 | 14 | 16 | 8 | 0 |

Fit a Poisson distribution to the data, and test its goodness of fit at the 5% level of significance. [8]

- 8 Optical lenses, produced by two suppliers, A and B , are measured for surface hardness. A random sample of 4 lenses from A has a mean hardness $\bar{x}_1 = 0.84$ and $s_1^2 = 0.009$, in appropriate units. The corresponding figures for a random sample of 6 lenses from B are $\bar{x}_2 = 1.06$ and $s_2^2 = 0.012$. The values of s_1^2 and s_2^2 are the unbiased estimates of their respective population variances. Obtain a 95% confidence interval for $\mu_2 - \mu_1$, where μ_1 and μ_2 are the mean hardnesses of lenses supplied by A and B respectively. [7]

State the assumptions you must make for your result to be valid. [1]

- 9 A chess player, X , plays a series of games against an opponent, Y . For each game, the probability that X wins is p , independently of the results of other games. Write down the probability, in terms of p and n , that X 's first win is in the n th game. [1]

The number of games up to and including X 's first win is denoted by N .

- (i) Write down $E(N)$ and $\text{Var}(N)$ in terms of p , and deduce $E(N^2)$. [4]
- (ii) Find $P(1 \leq N \leq a)$, where a is a positive integer. [2]
- (iii) If X plays 4 games against Y , show that the probability that this series of games contains at least 2 consecutive wins by X is $p^2(3 - 2p)$. [4]

- 10 A random sample of 10 members of a fitness course have their weights checked. Their weights before and after the course are given in the table.

| Member | A | B | C | D | E | F | G | H | I | J |
|--------------------|----|----|----|----|----|----|----|----|----|----|
| Weight before (kg) | 70 | 77 | 80 | 83 | 84 | 86 | 90 | 93 | 94 | 96 |
| Weight after (kg) | 72 | 74 | 78 | 83 | 80 | 87 | 85 | 94 | 90 | 95 |

The weight differences may be assumed to be observations from a normal distribution. Test whether the mean weight of the population of members has been reduced by this course,

- (i) using a 5% level of significance,
(ii) using a 2.5% level of significance.

[11]

- 11 Answer only **one** of the following two alternatives.

EITHER

A light elastic string of natural length 1.6 m has modulus of elasticity 100 N. One end of the string is attached to a fixed point. A particle of mass 4 kg is attached to the other end. Find the extension, e metres, of the string when the particle hangs freely in equilibrium at E . [2]

When the particle is in equilibrium at E , it is given a blow of impulse 20 N s vertically downwards. Show that, while the string remains taut, the motion of the particle is simple harmonic, and find the period and amplitude of this simple harmonic motion. [8]

Find also the time that elapses between the blow and the instant when the string first becomes slack. [4]

OR

The random variable X has probability density function given by

$$f(x) = \begin{cases} 2e^{-2x} & x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

- (i) Show that the distribution function of X is given by

$$F(x) = \begin{cases} 1 - e^{-2x} & x > 0, \\ 0 & \text{otherwise.} \end{cases} \quad [2]$$

- (ii) Show that $E(X) = \frac{1}{2}$ and find the median of X . [3]

The random variable Y is defined by $Y = X^2$.

- (iii) Find $P(Y < y)$ for $y > 0$, and obtain the probability density function of Y . [3]

- (iv) Express $E(Y)$ in terms of $E(X)$ and $\text{Var}(X)$, and deduce that $E(Y) = \frac{1}{2}$. [3]

- (v) Show that the median of Y is the square of the median of X . [3]

